QP1 - Oct. 29, 2020.

1) a) stat. state: $H\psi = E\cdot\psi$

$$H = \frac{p^2}{2m} \Rightarrow \psi = e^{ik\cdot x}$$

(5pt)

b) linear superposition:

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \cdot \int dk \cdot \phi(k) \cdot e^{ikx} \cdot e^{-iEt/\hbar}$$

coeff.s of linear superposition. (4pt)

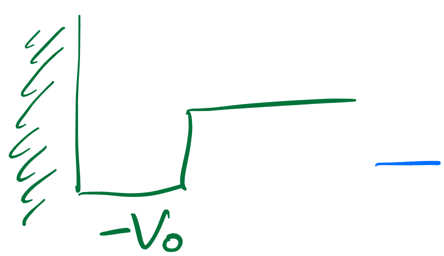
$E = \frac{\hbar^2 k^2}{2m}$ (3pt)

normalisation:

$$\int \psi^*\psi \, dx = \int \phi^*\phi \, dk = 1$$

(3pt)

c)



bound state with $-V_0 < E < 0$.

$x=0$ $\quad$ $x=x_0$

1) $\psi = 0$.

$\psi = Ce^{-lx} + D\cdot e^{+lx}$ 1)

$E = -\frac{\hbar^2 l^2}{2m}$ 1)

$\psi = A\cdot e^{ikx} + B\cdot e^{-ikx}$ 1)

$E - V_0 = \frac{\hbar^2 k^2}{2m}$ 1)

d) Normalisation : $D = 0$

$\Rightarrow \psi = C \cdot e^{-\ell x}$ 2pt)

ψ Continuous @ $x = 0$:

$A = -B \rightarrow \psi = 2i \cdot A \cdot \sin(kx)$ 4pt)

ψ and ψ' Continuous @ $x = x_0$:

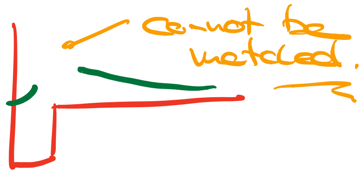
$$2i A \cdot \sin(k x_0) = C \cdot e^{-\ell x_0} \quad 2)$$
$$2ik \cdot A \cdot \cos(k x_0) = -\ell \cdot C \cdot e^{-\ell x_0} \quad 2)$$

e) Small x_0 $\simeq$ Taylor expansion :

$$2i A \cdot k \cdot x_0 = C \quad 2)$$
$$2i \cdot k \cdot A = -\ell \cdot C \quad 2)$$

$$\Rightarrow \quad x_0 = \frac{-1}{\ell} \quad 2)$$

so no bound state as x_0, ℓ are positive. 4pts)



2] a) Herm. operator:

$$\langle A | OB \rangle = \langle OA | B \rangle$$ 2pts)

take $A = B$ eigenvector with eigenvalue a:

$$\langle A | OA \rangle = a \cdot \langle A | A \rangle$$

$$= \langle OA | A \rangle = a^* \cdot \quad - \cdot$$ 3pts)

$$\Rightarrow \quad a = a^*$$

b) Reality:

3pts

* guaranteed for ∞-dim VS & Herm. oper. with discrete spectrum

* has to be assumed for -- 2pts

--- continuous ---.

c) yes, entangled: 3pts.

measuring left particle influences Right Result. 2pts.

3] a) $L_\pm = L_x \pm i L_y$ $\quad [L_x, L_y] = i\hbar L_z$

$$[L_\pm, L_z] = [L_x \pm i L_y, L_z]$$

$$= [L_x, L_z] \pm i [L_y, L_z]$$

$$= -i\hbar \cdot L_y \pm i \cdot i\hbar L_x$$

$$= \mp \hbar \cdot L_\pm$$ 5pts)

$$[L_\pm, L^2] = [L_x \pm i L_y, L_x^2 + L_y^2 + L_z^2]$$

$$= [L_x, L_y^2] + [L_x, L_z^2] \pm i \cdot [L_y, L_x^2] \pm i \, [L_y, L_z^2]$$

use $[L_x, L_y^2] = L_x L_y^2 - L_y^2 L_x$

$$= [L_x, L_y] L_y - L_y [L_y, L_x]$$

$$= i\hbar L_z L_y + i\hbar L_y L_z$$

$$[L_x, L_z^2] = L_x \cdot L_z^2 - L_z^2 \cdot L_x$$

$$= [L_x, L_z] L_z - L_z \cdot [L_z, L_x]$$

$$= -i\hbar \cdot L_y L_z - i\hbar \cdot L_z L_y .$$

cancel against each other.

$\Rightarrow$ $[\vec{L}, \vec{L}^2] = 0$ and

hence also $[L_\pm, \vec{L}^2] = 0$.

3pts)

b) if ψ is eigenstate of $\vec{L}^2$, L_z with eigenvalues λ, μ,

then $L_\pm \psi$ is also an eigenstate with eigenvalues $\lambda, \mu \pm \hbar$. 3pts 2pts

c). $\begin{matrix} l=1 \\ m=0 \end{matrix}$: $Y_1^0 = A \cdot \cos\Theta$.

act with L_+: 2pts)

$$Y_1^1 \sim +\hbar \cdot e^{+i\phi} \cdot \frac{\partial}{\partial\Theta} (A \cos\Theta)$$

$$\sim e^{+i\phi} \cdot \sin\Theta .$$

3pts)

ignoring normalisation.

d) $L_+ L_+$ on $Y_1^0 \Rightarrow 0$. 5pts)

explicitly:

$$\left(\frac{\partial}{\partial\theta} + \frac{i}{\tan\theta}\cdot\frac{\partial}{\partial\phi}\right) e^{+i\phi}\cdot\sin\theta$$

$$= e^{+i\phi}\cdot\cos\theta + i\cdot\cos\theta\cdot -i\cdot e^{+i\phi}$$

$$= 0. \quad \hbar.$$ 5pts)

e) $l_1 = 1$ plus $l_2 = 1$

$\Rightarrow$ total angular momentum ranges from

$$\underset{=2}{l_1 + l_2} \quad \cdots \quad \underset{=0}{l_1 - l_2}$$

$$\Rightarrow \quad l = 0, 1, 2.$$ 5pts).

(or you can say that

$$L^2 = l\cdot(l+1)\hbar^2$$

$$= 0, 2, 6 \cdot \hbar^2$$

for full ang. mom.).